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$K3 \times T^2/\mathbb{Z}_2$ orientifolds with fluxes, open string moduli and critical points

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Abstract

We extend the four–dimensional gauged supergravity analysis of type IIB vacua on $K3 \times T^2/\mathbb{Z}_2$ to the case where also D3 and D7 moduli, belonging to $\mathscr{N} = 2$ vector multiplets, are turned on. In this case, the overall special geometry does not correspond to a symmetric space, unless D3 or D7 moduli are switched off. In the presence of non–vanishing fluxes, we discuss supersymmetric critical points which correspond to Minkowski vacua, finding agreement with previous analysis. Finally, we point out that care is needed in the choice of the symplectic holomorphic sections of special geometry which enter the computation of the scalar potential.

1 Introduction

In the present note we generalise the four–dimensional supergravity analysis of [1] to the case where D3 and D7 brane moduli are turned on, together with 3–form fluxes. This problem seems of particular physical relevance in view of recent work on inflationary models [2]–[7], whose underlying scalar potential is in part predicted by the mechanism of supergravity breaking, which is at work in some string compactifications in the presence of fluxes [8]–[17].

From the point of view of the four–dimensional $\mathscr{N}=2$ effective supergravity, open string moduli, corresponding to D7 and D3–brane positions along T^2, form an enlargement of the vector multiplet moduli–space which is locally described, in absence of open–string moduli, by [1]:

$$\left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)_s \times \left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)_t \times \left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)_u \,, \tag{1.1}$$

where s, t, u denote the scalars of the vector multiplets containing the $K3$–volume and the R–R $K3$–volume–form, the T^2–complex structure, and the IIB axion–dilaton system, respectively:

$$\begin{aligned} s &= C_{(4)} - \mathrm{i}\,\mathrm{Vol}(K_3), \\ t &= \frac{g_{12}}{g_{22}} + \mathrm{i}\,\frac{\sqrt{\det g}}{g_{22}}\,, \\ u &= C_{(0)} + \mathrm{i}\,e^{\phi}\,, \end{aligned} \tag{1.2}$$

where the matrix g denotes the metric on T^2.

When D7–branes moduli are turned on, what is known is that $\mathrm{SU}(1,1)_s$ acts as an electric–magnetic duality transformation [18] both on the bulk and D7–brane vector field–strengths, while the $\mathrm{SU}(1,1)_u$ acts as an electric–magnetic duality transformation on the D3–vector field–strengths. Likewise the bulk vectors transform perturbatively under $\mathrm{SU}(1,1)_u \times \mathrm{SU}(1,1)_t$ while the D3–brane vectors do not transform under $\mathrm{SU}(1,1)_s \times \mathrm{SU}(1,1)_t$ and the D7–brane vectors do not transform under $\mathrm{SU}(1,1)_u \times \mathrm{SU}(1,1)_t$.

All this is achieved starting from the following trilinear prepotential of special geometry:

$$\mathscr{F}(s,t,u,x^k,y^r) \,=\, stu - \frac{1}{2}\,s\,x^k x^k - \frac{1}{2}\,u\,y^r y^r\,, \tag{1.3}$$

where x^k and y^r are the positions of the D7 and D3–branes along T^2 respectively, $k = 1,\ldots,n_7$, $r = 1,\ldots,n_3$, and summation over repeated indices is understood. This prepotential is unique in order to preserve the shift–symmetries of the s, t, u bulk complex fields up to terms which only depend on x and y.

The above prepotential gives the correct answer if we set either all the x^k or all the y^r to zero. In this case the special geometry describes a symmetric space:

$$\left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)_s \times \frac{\mathrm{SO}(2,2+n_7)}{\mathrm{SO}(2)\times\mathrm{SO}(2+n_7)}\,,\quad \text{for } y^r=0\,, \tag{1.4}$$

$$\left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)_u \times \frac{\mathrm{SO}(2,2+n_3)}{\mathrm{SO}(2)\times\mathrm{SO}(2+n_3)}\,,\quad \text{for } x^k=0\,. \tag{1.5}$$

For both x and y non–vanishing, the complete Kähler manifold (of complex dimension $3+n_3+n_7$) is no longer a symmetric space even if it still has $3+n_3+n_7$ shift symmetries[1].

Note that for $x^k=0$ the manifold is predicted as a truncation of the manifold describing the moduli–space of $T^6/\mathbb{Z}_2$ $\mathscr{N}=4$ orientifold in the presence of D3–branes. The corresponding symplectic embedding was given in [20]. For $y^r=0$ the moduli–space is predicted by the way $\mathrm{SU}(1,1)_s$ acts on both bulk and D7 vector fields. Upon compactification of Type IIB theory on T^2, the D7–brane moduli are insensitive to the further $K3$ compactification and thus their gravity coupling must be the same as for vector multiplets coupled to supergravity in $D=8$. Indeed if $2+n$ vector multiplets are coupled to $\mathscr{N}=2$ supergravity in $D=8$, their non–linear σ–model is [21],[22]:

$$\frac{\mathrm{SO}(2,2+n)}{\mathrm{SO}(2)\times\mathrm{SO}(2+n)}\times\mathbb{R}^+\,. \tag{1.6}$$

Here $\mathbb{R}^+$ denotes the volume of T^2 and the other part is the second factor in (1.4). Note that in $D=8$, $\mathscr{N}=2$ the R–symmetry is U(1) which is the U(1) part of the $D=4$, $\mathscr{N}=2$ U(2) R–symmetry. The above considerations prove eq. (1.4).

Particular care is needed [23] when the effective supergravity is extended to include gauge couplings, as a result of turning on fluxes in the IIB compactification [8].

The reason is that the scalar potential depends explicitly on the symplectic embedding of the holomorphic sections of special geometry, while the Kähler potential, being symplectic invariant, does not. In fact, even in the analysis without open string moduli [1], it was crucial to consider a Calabi–Visentini basis where the SO(2, 2) linearly acting symmetry on the bulk fields was $\mathrm{SU}(1,1)_u\times\mathrm{SU}(1,1)_t$ [24],[25].

In the case at hand, the choice of symplectic basis is the one which corresponds to the Calabi–Visentini basis for $y^r=0$, with the $\mathrm{SU}(1,1)_s$ acting as an electric–magnetic duality transformation [1], but it is not such basis for the D3–branes even if the $x^k=0$. Indeed, for $x^k=0$, we must reproduce the mixed basis used for the $T^6/\mathbb{Z}_2$ orientifold [26],[27] in the presence of D3–branes found in [20].

[1]The prepotential in eq. (1.3) actually corresponds to the homogeneous not symmetric spaces called $L(0,n_7,n_3)$ in [19]. We thank A. van Proeyen for a discussion on this point.

We note in this respect, that the choice of the symplectic section made in [7] does not determine type IIB vacua with the 3–form fluxes turned on. It does not correspond in fact to the symplectic embedding discussed in [1], [20] and [23]. The problem arises already in the absence of branes. Indeed in [7] the type IIB duality group $\mathrm{SU}(1,1)_u$, which is associated in their notation to the modulus S, has a non–perturbative action on the bulk vector fields while this action should be perturbative. As a consequence of this, in [7] a potential was found that does not stabilise the axion–dilaton field, which is in contradiction with known results [28][2].

2 $\mathscr{N}=2$ and $\mathscr{N}=1$ supersymmetric cases.

2.1. $\mathscr{N}=2$ gauged supergravity

We consider the gauging of $\mathscr{N}=2$ supergravity with a special geometry given by eq. (1.3). Let us briefly recall the main formulae of special Kähler geometry. The geometry of the manifold is encoded in the holomorphic section $\varOmega=(X^\varLambda,\, F_\varSigma)$ which, in the *special coordinate* symplectic frame, is expressed in terms of a prepotential $\mathscr{F}(s,t,u,x^k,y^r)=F(X^\varLambda)/(X^0)^2=\mathscr{F}(X^\varLambda/X^0)$, as follows:

$$\varOmega=(X^\varLambda,\, F_\varLambda=\partial F/\partial X^\varLambda)\,. \tag{2.7}$$

In our case $\mathscr{F}$ is given by eq. (1.3). The Kähler potential K is given by the symplectic invariant expression:

$$K=-\log\left[\mathrm{i}(\overline{X}^\varLambda F_\varLambda-\overline{F}_\varLambda X^\varLambda)\right]\,. \tag{2.8}$$

In terms of K the metric has the form $g_{i\bar\jmath}=\partial_i\partial_{\bar\jmath}K$. The matrices $U^{\varLambda\varSigma}$ and $\overline{\mathscr{N}}_{\varLambda\varSigma}$ are respectively given by:

$$\begin{aligned} U^{\varLambda\varSigma} &= e^K\,\mathscr{D}_iX^\varLambda\mathscr{D}_{\bar\jmath}\overline{X}^\varSigma\, g^{i\bar\jmath}=-\frac{1}{2}\,\mathrm{Im}(\mathscr{N})^{-1}-e^K\,\overline{X}^\varLambda X^\varSigma\,,\\ \overline{\mathscr{N}}_{\varLambda\varSigma} &= \hat{h}_{\varLambda|I}\circ(\hat{f}^{-1})^I{}_\varSigma\,,\ \text{where}\ \ \hat{f}_I^\varLambda=\begin{pmatrix}\mathscr{D}_iX^\varLambda\\ \overline{X}^\varLambda\end{pmatrix}\ ;\ \ \hat{h}_{\varLambda|I}=\begin{pmatrix}\mathscr{D}_iF_\varLambda\\ \overline{F}_\varLambda\end{pmatrix}\,. \end{aligned} \tag{2.9}$$

For our choice of $\mathscr{F}$, K has the following form:

$$K=-\log\left[-8\,(\mathrm{Im}(s)\,\mathrm{Im}(t)\mathrm{Im}(u)-\frac{1}{2}\,\mathrm{Im}(s)\,(\mathrm{Im}(x)^i\,)^2-\frac{1}{2}\,\mathrm{Im}(u)\,(\mathrm{Im}(y)^r\,)^2)\right]\,, \tag{2.10}$$

with $\mathrm{Im}(s)<0$ and $\mathrm{Im}(t)$, $\mathrm{Im}(u)>0$ at $x^k=y^r=0$. The components $X^\varLambda$, $F_\varSigma$ of the symplectic section which correctly describe our problem, are chosen by performing a

[2] Actually, in a revised version of [7], agreement with our results is found

constant symplectic change of basis from the one in (2.7) given in terms of the prepotential in eq. (1.3). The symplectic matrix is

$$\begin{pmatrix} A & -B \\ B & A \end{pmatrix}, \tag{2.11}$$

with

$$\begin{aligned} A &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{2} \end{pmatrix}, \\ B &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{2.12}$$

The rotated symplectic sections then become

$$\begin{aligned} X^0 &= \frac{1}{\sqrt{2}} (1 - t\,u + \frac{(x^k)^2}{2}), \\ X^1 &= -\frac{t+u}{\sqrt{2}}, \\ X^2 &= -\frac{1}{\sqrt{2}} (1 + t\,u - \frac{(x^k)^2}{2}), \\ X^3 &= \frac{t-u}{\sqrt{2}}, \\ X^k &= x^k, \\ X^a &= y^r, \\ F_0 &= \frac{s\,\left(2 - 2\,t\,u + (x^k)^2\right) + u\,(y^r)^2}{2\,\sqrt{2}}, \\ F_1 &= \frac{-2\,s\,(t+u) + (y^r)^2}{2\,\sqrt{2}}, \\ F_2 &= \frac{s\,\left(2 + 2\,t\,u - (x^k)^2\right) - u\,(y^r)^2}{2\,\sqrt{2}}, \\ F_3 &= \frac{2\,s\,(-t+u) + (y^r)^2}{2\,\sqrt{2}}, \\ F_i &= -s\,x^k, \\ F_a &= -u\,y^r. \end{aligned} \tag{2.13}$$

Note that, since $\partial X^\Lambda/\partial s = 0$ the new sections do not admit a prepotential, and the no–go theorem on partial supersymmetry breaking [29] does not apply in this case. As in [1], we limit ourselves to gauge shift–symmetries of the quaternionic manifold of the $K3$ moduli–space. Other gaugings which include the gauge group on the brane will be considered elsewhere.

2.2. $\mathscr{N} = 2$ supersymmetric critical points

In the sequel we limit our analysis to critical points in flat space. The $\mathscr{N} = 2$ critical points demand $\mathscr{P}^x_\Lambda = 0$. This equation does not depend on the special geometry and its solution is the same as in [1], i.e. $g_2,\, g_3 \neq 0$, $g_0 = g_1 = 0$ and $e^m_a = 0$ for $a = 1, 2$, were the Killing vectors gauged by the fields A^2_μ and A^3_μ are constants and their non–vanishing components are $k^u_2 = g_2$ along the direction $q^u = C^{a=1}$ and $k^u_3 = g_3$ along the direction $q^u = C^{a=2}$. The 22 fields C^m, C^a, $m = 1, 2, 3$ and $a = 1, \dots, 19$ denote the Peccei–Quinn scalars. The vanishing of the hyperino–variation further demands:

$$k^u_\Lambda X^\Lambda = 0 \;\Rightarrow\; X^2 = X^3 = 0 \;\Leftrightarrow\; t = u\,,\;\; 1 + t^2 = \frac{(x^k)^2}{2}\,. \tag{2.14}$$

Hence for $\mathscr{N} = 2$ vacua the D7 and D3–brane positions are still moduli while the axion–dilaton and T^2 complex structure are stabilised.

2.3. $\mathscr{N} = 1$ supersymmetric critical points

The $\mathscr{N} = 1$ critical points in flat space studied in [1] were first obtained by setting $g_0,\, g_1 \neq 0$ and $g_2 = g_3 = 0$, with $k^u_0 = g_0$ along the direction $q^u = C^{m=1}$ and $k^u_1 = g_1$ along the direction $q^u = C^{m=2}$.

Constant Killing spinors. By imposing $\delta_{\epsilon_2} f = 0$ for the variations of the fermionic fields f we get the following:
From the hyperino variations:

$$\begin{aligned} \delta_{\epsilon_2} \xi^{Aa} &= 0 \;\Rightarrow\; e^a_m = 0 \; m = 1, 2\,;\; a = 1, \dots, 19 \\ \delta_{\epsilon_2} \xi^{A} &= 0 \;\Rightarrow\; \text{vanishing of the gravitino variation} \end{aligned} \tag{2.15}$$

The gravitino variation vanishes if:

$$S_{22} = -g_0\, X^0 + \mathrm{i}\, g_1\, X^1 \tag{2.16}$$

From the gaugino variations we obtain:

$$\delta_{\epsilon_2} (\lambda^{\bar{\imath}})_A = 0 \;\Rightarrow\; e^{\frac{K}{2}}\, \mathscr{P}^x_\Lambda\, (\partial_i X^\Lambda + (\partial_i K)\, X^\Lambda)\, \sigma^x_{A2} \,=\, 0\,, \tag{2.17}$$

the second term (with $\partial_i K$) gives a contribution proportional to the gravitino variation while the first term, for $i = u,\, t,\, x^k$ respectively gives:

$$\begin{aligned} -\, g_0\, \partial_u X^0 + \mathrm{i}\, \partial_u X^1 &= 0\,, \\ -g_0\, \partial_t X^0 + \mathrm{i}\, \partial_t X^1 &= 0\,, \\ -g_0\, \partial_{x^k} X^0 &= 0\,, \end{aligned} \tag{2.18}$$

for $i = y^r$ the equation is identically satisfied. From the last equation we get $x^k = 0$ and the other two, together with $S_{22} = 0$ give $u = t = \mathrm{i}$, $g_0 = g_1$.

So we see that for $\mathscr{N} = 1$ vacua the D7–brane coordinates are frozen while the D3–brane coordinates remain moduli. This agrees with the analysis of [28]. If $g_0 \neq g_1$ the above solutions give critical points with vanishing cosmological constant but with no supersymmetry left.

More general $\mathscr{N} = 1, 0$ vacua can be obtained also in this case by setting $g_2,\, g_3 \neq 0$. The only extra conditions coming from the gaugino variations for $\mathscr{N} = 1$ vacua is that $e_m^{a=1,2} = 0$. This eliminates from the spectrum two extra metric scalars $e_3^{a=1,2}$ and the $C_{a=1,2}$ axions. These critical points preserve $\mathscr{N} = 1$ or not depending on whether $|g_0| = |g_1|$ or not.

We can describe the $\mathscr{N} = 1 \to \mathscr{N} = 0$ transition with an $\mathscr{N} = 1$ no–scale supergravity [30, 31] based on a constant superpotential and a non–linear sigma–model which is

$$\frac{\mathrm{U}(1, 1 + n_3)}{\mathrm{U}(1) \times \mathrm{U}(1 + n_3)} \times \frac{\mathrm{SO}(2, 18)}{\mathrm{SO}(2) \times \mathrm{SO}(18)}\,, \tag{2.19}$$

where the two factors come from vector multiplets and hypermultiplets, respectively. This model has vanishing scalar potential, reflecting the fact that there are not further scalars becoming massive in this transition [1]. We further note that any superpotential $W(y)$ for the D3 brane coordinates would generate a potential [32] term

$$e^K\, K^{y\bar{y}}\, \partial_y W \partial_{\bar{y}} \bar{W}\,, \tag{2.20}$$

which then would require the extra condition $\partial_y W = 0$ for a critical point with vanishing vacuum energy.

The residual moduli space of K3 metrics at fixed volume is locally given by

$$\frac{\mathrm{SO}(1, 17)}{\mathrm{SO}(17)}\,. \tag{2.21}$$

We again remark that we have considered vacua with vanishing vacuum energy. We do not consider here the possibility of other vacua with non–zero vacuum energy, as i.e. in [7].

3 The potential

The general form of the $\mathscr{N}=2$ scalar potential is:

$$V = 4\,e^{K} h_{uv} k^u_\Lambda k^v_\Sigma\, X^\Lambda \overline{X}^\Sigma + e^K g_{i\bar{\jmath}}\, k^i_\Lambda k^{\bar{\jmath}}_\Sigma\, X^\Lambda \overline{X}^\Sigma + e^K (U^{\Lambda\Sigma} - 3\,e^K\, X^\Lambda \overline{X}^\Sigma) \mathscr{P}^x_\Lambda\, \mathscr{P}^x_\Sigma \quad (3.22)$$

where the second term is vanishing for abelian gaugings. Here h_{uv} is the quaternionic metric and k^u_Λ the quaternionic Killing vector of the hypermultiplet σ–model.

The scalar potential, at the extremum of the e^a_m scalars, has the following form[3]:

$$\begin{aligned} V \;&=\; 4\,e^{2\,\varphi}\,e^K\,\Bigg[\sum_{\Lambda=0}^{3}(g_\Lambda)^2\,|X^\Lambda|^2 + \frac{1}{2}\,(g_0^2+g_1^2)(t-\bar{t})\left((u-\bar{u}) - \frac{1}{2}\,\frac{(x^k-\bar{x}^k)^2}{(t-\bar{t})}\right) \\ &\quad + \frac{(y^r-\bar{y}^r)^2}{8\,(s-\bar{s})(u-\bar{u})}\left(g_0^2\,(\bar{u}x^k - \bar{x}^k u)^2 + g_1^2\,(x^k-\bar{x}^k)^2\right)\Bigg]\,. \end{aligned} \quad (3.23)$$

From the above expression we see that in the $\mathscr{N}=2$ case, namely for $g_0 = g_1 = 0$, the potential depends on y^r only through the factor e^K and vanishes identically in y^r for the values of the t, u scalars given in (2.14), for which $X^2 = X^3 = 0$. If g_0 or g_1 are non–vanishing ($\mathscr{N}=1, 0$ cases) the extremisation of the potential with respect to x^k, namely $\partial_{x^k} V = 0$ fixes $x^k = 0$. For $x^k = 0$ the potential depends on y^r only through the factor e^K and vanishes identically in y^r for $t = u = \mathrm{i}$.

3.1. Generalised gauging

From a four–dimensional supergravity point of view we could consider a generalisation of the previous gauging in which also the D7 and D3–brane vectors A^i_μ, A^r_μ are used to gauge translational isometries along the directions C^a. We can choose for simplicity to turn on couplings for all the n_3 D3–brane vectors and n_7 D7–brane vectors with the constraint $n_3 + n_7 \leq 17$. These new couplings do not have an immediate interpretation in terms of string compactification with fluxes. The constant Killing vectors are $k^u_\Lambda = g^k_4$, $\Lambda = 3 + k$, $k = 1, \ldots, n_7$, along the direction $q^u = C^{a=3,\ldots,2+n_7}$ and $k^u_\Lambda = g^r_5$, $\Lambda = 3 + n_7 + r$, $r = 1, \ldots, n_3$, along the direction $q^u = C^{a=3+n_7,\ldots,2+n_3+n_7}$. The expression of the new potential, at the extremum of the e^m_a scalars, is a trivial extension of eq. (3.23):

$$\begin{aligned} V \;&=\; 4\,e^{2\,\varphi}\,e^K\,\Bigg[\sum_{\Lambda=0}^{3}(g_\Lambda)^2\,|X^\Lambda|^2 + \sum_{k=1}^{n_7}(g^k_4)^2\,|X^{3+k}|^2 + \sum_{r=1}^{n_3}(g^r_5)^2\,|X^{3+n_7+r}|^2 \\ &\quad + \frac{1}{2}\,(g_0^2+g_1^2)(t-\bar{t})\left((u-\bar{u}) - \frac{1}{2}\,\frac{(x^k-\bar{x}^k)^2}{(t-\bar{t})}\right) \\ &\quad + \frac{(y^r-\bar{y}^r)^2}{8\,(s-\bar{s})(u-\bar{u})}\left(g_0^2\,(\bar{u}x^k - \bar{x}^k u)^2 + g_1^2\,(x^k-\bar{x}^k)^2\right)\Bigg]\,. \end{aligned} \quad (3.24)$$

[3] Note that there is a misprint in eq. (5.1) of ref. [1]. The term $e^{2\phi}\,e^{\tilde{K}}\,g_0\,g_1(X_0\bar{X}_1 + X_1\bar{X}_0)$ is actually absent

As far as supersymmetric vacua are concerned, from inspection of the fermion shifts it is straightforward to verify that the existence of a constant Killing spinor always requires X^2, X^3, X^{3+k}, $X^{3+n_7+r} = 0$ which implies $x^k = y^r = 0$ and $t = u = \mathrm{i}$. As before we have $\mathcal{N} = 2$ if $g_0 = g_1 = 0$, $\mathcal{N} = 1$ if $g_0 = g_1 \neq 0$ and $\mathcal{N} = 0$ otherwise. The $\mathcal{N} = 0$ flat vacuum is also defined by the conditions $x = y = 0$ and $t = u = \mathrm{i}$, as it can be verified from eq. (3.24). The presence of non–vanishing g_4^k and g_5^r couplings therefore fixes the positions of the branes along T^2 to $x^k = y^r = 0$ in all the relevant cases.

4 Conclusions

The present investigation allows us to study in a fairly general way the potential for the 3–form flux compactification, in presence of both bulk and open string moduli. In absence of fluxes the D3, D7 dependence of the Kähler potential is rather different since this moduli couple in different ways to the bulk moduli.

Moreover, in the presence of 3–form fluxes which break $\mathcal{N} = 2 \to \mathcal{N} = 1, 0$ the D7 moduli are stabilised while the D3 moduli are not. For small values of the coordinates x^k, y^r the dependence of their kinetic term is (for $u = t = \mathrm{i}$), $-(\partial_\mu \bar{y}^r \partial^\mu y^r)/\mathrm{Im}(s)$ for the D3–brane moduli, and $-(\partial_\mu \bar{x}^k \partial^\mu x^k)$ for the D7–brane moduli. This is in accordance with the suggestion of [2]. Note that the above formulae, at $x = 0$, $u = t = \mathrm{i}$ are true up to corrections $O(\frac{Im(y)^2}{Im(s)})$, since y and s are moduli even in presence of fluxes. The actual dependence of these terms on the compactification volume is important in order to further consider models for inflatons where the terms in the scalar potential allow to stabilise the remaining moduli.

Finally, we have not considered here the gauging of compact gauge groups which exist on the brane world–volumes. This is, for instance, required [33, 34, 7] in models with hybrid inflation [35]. This issue will be considered elsewhere.

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Appendix A Some relevant formulae.

We are interested in gauging the 22 translations in the coset $\mathrm{SO}(4,20)/(\mathrm{SO}(3,19) \times \mathrm{O}(1,1))$. Let us denote by L the coset representative of $\mathrm{SO}(3,19)/\mathrm{SO}(3) \times \mathrm{SO}(19)$. It will be written in the form:

$$L = \begin{pmatrix} (1+\mathbf{e}\,\mathbf{e}^T)^{\frac{1}{2}} & -\mathbf{e} \\ -\mathbf{e}^T & (1+\mathbf{e}^T\mathbf{e})^{\frac{1}{2}} \end{pmatrix} \tag{A.1}$$

where $\mathbf{e} = \{e^m{}_a\}$, $\mathbf{e}^T = \{e^a{}_m\}$, $m = 1,2,3$ and $a = 1,\ldots,19$, are the coordinates of the manifold. The 22 nilpotent Peccei–Quinn generators are denoted by $\{Z_m,\, Z_a\}$ and the gauge generators are:

$$t_\Lambda = f^m{}_\Lambda Z_m + h^a{}_\Lambda Z_a \tag{A.2}$$

the corresponding Killing vectors have non vanishing components: $k^m_\Lambda = f^m{}_\Lambda$ and $k^a_\Lambda = h^a{}_\Lambda$. The moment maps are:

$$\mathscr{P}^x_\Lambda = \sqrt{2}\,\left(e^\varphi\,(L^{-1})^x{}_m\, f^m{}_\Lambda + e^\varphi\,(L^{-1})^x{}_a\, h^a{}_\Lambda\right) \tag{A.3}$$

where φ is the T^2 volume modulus [1]: $e^{-2\varphi} = \mathrm{Vol}(T^2)$ and $x = 1,2,3$. The metric along the Peccei–Quinn directions $I = (m,a)$ is:

$$h_{IJ} = e^{2\varphi}\,(\delta_{IJ} + 2\, e^a{}_I e^a{}_J) \tag{A.4}$$

The potential has the following form:

$$\begin{aligned} V \;\; = \;\; & 4\, e^{2\varphi}\,(f^m{}_\Lambda\, f^m{}_\Sigma + 2\, e^a{}_m e^a{}_n\, f^m{}_\Lambda\, f^n{}_\Sigma + h^a{}_\Lambda\, h^a{}_\Sigma)\ \bar{L}^\Lambda\, L^\Sigma \\ & +2\, e^{2\varphi}\,\left(U^{\Lambda\Sigma} - 3\,\bar{L}^\Lambda\, L^\Sigma\right)\,\Big(f^m{}_\Lambda\, f^m{}_\Sigma + e^a{}_m e^a{}_n\, f^m{}_\Lambda\, f^n{}_\Sigma \\ & +2\,[(1+\mathbf{e}\,\mathbf{e}^T)^{\frac{1}{2}}]^n{}_m e^n{}_a\, f^m{}_{(\Lambda}\, h^a{}_{\Sigma)} + e^n{}_a e^n{}_b\, h^a{}_\Lambda\, h^b{}_\Sigma\Big)\ . \end{aligned} \tag{A.5}$$

In all the models we consider, at the extremum point of the potential in the special Kähler manifold the following condition holds: $\left(U^{\Lambda\Sigma} - 3\,\bar{L}^\Lambda\, L^\Sigma\right)_{|0}\, f^m{}_{(\Lambda}\, h^a{}_{\Sigma)} = 0$. As a consequence of this, as it is clear from (A.5), the potential in this point depends on the metric scalars e^m_a only through quadratic terms in the combinations $e^m{}_a\, h^a{}_\Lambda$ and $e^a{}_m\, f^m{}_\Lambda$. Therefore V is extremised with respect to the e^m_a scalars once we restrict ourselves to the moduli defined as follows:

$$\text{moduli:} \qquad e^m{}_a\, h^a{}_\Lambda = e^a{}_m\, f^m{}_\Lambda = 0\,. \tag{A.6}$$

The vanishing of the potential implies

$$\left(U^{\Lambda\Sigma} -\ \bar{L}^\Lambda\, L^\Sigma\right)_{|0}\, f^m{}_{(\Lambda}\, f^m{}_{\Sigma)} + 2\,\left(\bar{L}^\Lambda\, L^\Sigma\right)_{|0}\, h^a{}_{(\Lambda}\, h^a{}_{\Sigma)} = 0\,. \tag{A.7}$$

Furthermore, one may notice that, as in [1], the following relations hold in all the models under consideration:

$$\left(U^{\Lambda\Sigma} - \bar{L}^{\Lambda} L^{\Sigma}\right)_{|0} f^{m}{}_{(\Lambda} f^{m}{}_{\Sigma)} = \left(\bar{L}^{\Lambda} L^{\Sigma}\right)_{|0} h^{a}{}_{(\Lambda} h^{a}{}_{\Sigma)} = 0\,. \tag{A.8}$$

Our analysis is limited to the case in which the only non–vanishing f and h constants are:

$$f^{1}{}_{0} = g_0 \;;\;\; f^{2}{}_{1} = g_1 \;;\;\; h^{1}{}_{2} = g_2 \;;\;\; h^{2}{}_{3} = g_3 \;;\;\; h^{2+k}{}_{3+k} = g_4^k \;;\;\; h^{2+n_7+r}{}_{3+n_7+r} = g_5^r\,. \tag{A.9}$$

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